



## 24<sup>th</sup> BALKAN MATHEMATICAL OLYMPIAD

Rhodes, Hellas, 28 April 2007

### Problem 1.

Let  $ABCD$  be a convex quadrilateral with  $AB = BC = CD$ ,  $AC \neq BD$  and let  $E$  be the intersection point of its diagonals. Prove that  $AE = DE$  if and only if  $\angle BAD + \angle ADC = 120^\circ$ .

### Problem 2.

Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) + y) = f(f(x) - y) + 4f(x)y, \text{ for any } x, y \in \mathbb{R}.$$

### Problem 3.

Find all positive integers  $n$  such that there is a permutation  $\sigma$  of the set  $\{1, 2, \dots, n\}$  for which  $\sqrt{\sigma(1) + \sqrt{\sigma(2) + \sqrt{\dots + \sqrt{\sigma(n)}}}}$  is a rational number.

*Note:* A permutation of the set  $\{1, 2, \dots, n\}$  is a one-to-one function of this set to itself.

### Problem 4.

For a given positive integer  $n > 2$ , let  $C_1, C_2, C_3$  be the boundaries of three convex  $n$ -gons in the plane such that  $C_1 \cap C_2, C_2 \cap C_3, C_3 \cap C_1$  are finite. Find the maximum number of points of the set  $C_1 \cap C_2 \cap C_3$ .

*Time allowed 4 hours and 30 minutes*  
*Each problem is worth 10 points.*